NAME: ______________________ STUDENT #__________

AESB2320, 2016-17
Part 2 Examination - 19 April, 2017

<u>Turn in this exam with your answer sheet.</u>

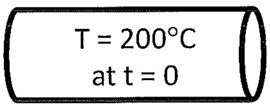
Write your solutions *on your answer sheet*, not here. In all cases ***show your work***.
To avoid any possible confusion,
state the equation numbers and figure numbers of equations and figures you use along with the text you are using (BSL2 or BSLK).
Beware of unnecessary information in the problem statement.

1. A cylindrical wire of radius R, like that in BSL 1st edition (BSL1) section 9.2, is heated by generation, but in this case the generation rate varies with radial position:
   $S_e = A\,r^C$
   where r is radial position and A and C are constants. At the outer radius, there is convective heat transfer with a surrounding fluid at temperature T_o, with heat-transfer coefficient h. What is the *last* equation in the derivation of BSL1 Section 9.2 that can be applied directly to this problem? Write that equation (*on your answer sheet, not here*) as

   BSL1 Eq. 9.2-__

   The relevant pages of BSL1 are appended to the end of this exam.
   (10 points)

2. A cylinder, 5 cm in diameter and 20 cm long, is initially at a uniform temperature of 200°C. It has properties given below. It is allowed to cool, surrounded by air at 20°C. Both flat surfaces are perfectly insulated. The heat-transfer coefficient at the cylinder surface is $h = 50\ \mathrm{W/(m^2K)}$. For simplicity, for parts (a) and (b), *assume that the cylinder is at a uniform temperature at all times as it cools.*



surrounded by air at 20°C

   a. Derive a formula for the heat flux (heat transfer per unit area) from the cylinder to the surrounding air as a function of time, with properties plugged in.
   b. Derive a formula for the temperature of the cylinder as a function of time, with numerical values of properties plugged in.
   c. In reality, of course, the assumption that the cylinder is at an absolutely uniform temperature as it cools is an approximation; there is a small temperature gradient (dq_r/dr) within the cylinder. There is concern that if the temperature gradient in the cylinder near its surface is too great, the cylinder may crack. *When* would the temperature gradient at the cylinder surface be the greatest? Calculate the temperature gradient at the surface at this time as a function of the parameters of this problem. For the purposes of this part, assume that your results for parts (a) and (b) are correct; you don't need to recompute those answers. (If you don't get this part, don't spend too long on it. Most of the credit is for parts (a) and (b).)

<u>Properties of solid</u>

$k = 35\ \mathrm{W/(m\ K)}$ $C_p = 130\ \mathrm{J/(kg\ K)}$ $\rho = 11{,}000\ \mathrm{kg/m^3}$

(40 points)

3. Rocky wants to measure the heat-transfer coefficient for convective heat transfer between a cold metal sphere and surrounding water, by measuring the temperature of the sphere. Of course therefore he wants convective heat transfer to the surface to control the heat-transfer process. He has a choice between two metals for the sphere, A and B, with properties listed below. Does it matter which metal he chooses? If so, which metal should he choose? Briefly justify your answer.

Properties of metal A

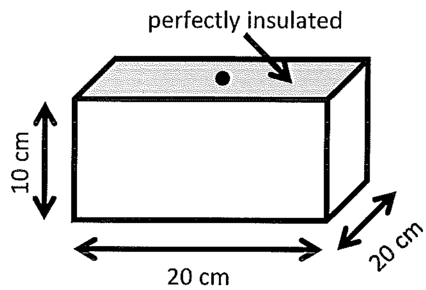
c_p = 129.9 J/(kg K)   k = 293 W/(m K) ρ = 19,320 kg/m^3

Properties of metal B

c_p = 126 J/(kg K)   k = 26 W/(m K) ρ = 11300 kg/m^3

(15 points)

4. A rectangular solid is 20 cm x 20 cm x 10 cm, is initially at 0°C. One side is perfectly insulated, as shown. Starting at time t = 0, the five other surfaces are changed to 100°C and maintained at that temperature.



   a. What is the temperature at a location in the middle of the perfectly insulated side (shown as black dot in figure) after 5 min.?
   b. What is the temperature at that same location after 8 min.?

(25 points)

Properties of solid

ρ = 7820 kg/m^3 c_p = 461 J/(kg K)   k = 23 W/(m K)

5. Based on your answer to problem 4, answer the following question: Suppose a solid of that same shape and properties were initially at 0°C. Starting at time t = 0 the five non-insulated surfaces are changed to, and maintained at, 100°C. Three minutes later those surfaces are changed back to 0°C and maintained at that temperature. What is the temperature at the location indicated, 5 minutes after the second change, i.e. 8 minutes after the first temperature change?

   To answer this question, and help me with grading, write out a formula *with all the numbers plugged in*, and then compute the final answer from this formula. If you weren't able to solve problem 4, just state clearly what answers you are assuming for parts 4(a) and 4(b) and state your answer in terms of those numbers.

(10 points)

Note on problem 2: Some students were confused by the meaning of *gradient* in part (c); *gradient* means derivative in space (dq_r/dr), not in time. The problem implies this by stating that the gradient is nonzero because temperature is not absolutely uniform. I've added that clarification here, but it was not there on the day of the exam.

dependence of either the thermal or electrical conductivity need be considered. The surface of the wire is maintained at temperature T_0. We now show how one can determine the radial temperature distribution within the heated wire.

For the energy balance we select as the system a cylindrical shell of thickness Δr and length L. (See Fig. 9.2–1.) The various contributions to the energy balance are

rate of thermal energy in across cylindrical surface at r	$(2\pi rL)(q_r\|_r)$	(9.2–2)
rate of thermal energy out across cylindrical surface at $r + \Delta r$	$(2\pi(r + \Delta r)L)(q_r\|_{r+\Delta r})$	(9.2–3)
rate of production of thermal energy by electrical dissipation	$(2\pi r\,\Delta rL)S_e$	(9.2–4)

The notation q_r means "flux of energy in the r-direction," and $|_r$ means "evaluated at r." Note that we take "in" and "out" to be in the positive r-direction.

We now substitute these three expressions into Eq. 9.1–1. Division by $2\pi L\,\Delta r$ and taking the limit as Δr goes to zero gives

$$\left\{\lim_{\Delta r\to 0}\frac{(rq_r)|_{r+\Delta r} - (rq_r)|_r}{\Delta r}\right\} = S_e r \tag{9.2–5}$$

The expression within braces is just the first derivative of rq_r with respect to r, so that Eq. 9.2–5 becomes

$$\frac{d}{dr}(rq_r) = S_e r \tag{9.2–6}$$

This is a first-order ordinary differential equation for the energy flux, which may be integrated to give

$$q_r = \frac{S_e r}{2} + \frac{C_1}{r} \tag{9.2–7}$$

The integration constant C_1 must be zero because of the boundary condition

B.C. 1: at $r = 0$ q_r is not infinite (9.2–8)

Hence the final expression for the energy flux distribution is

$$\boxed{q_r = \frac{S_e r}{2}} \tag{9.2–9}$$

This states that the heat flux increases linearly with r.

We now substitute Fourier's law (see Eq. 8.1–2) in the form $q_r = -k(dT/dr)$ into Eq. 9.2–9 to obtain

$$-k\frac{dT}{dr} = \frac{S_e r}{2} \tag{9.2–10}$$

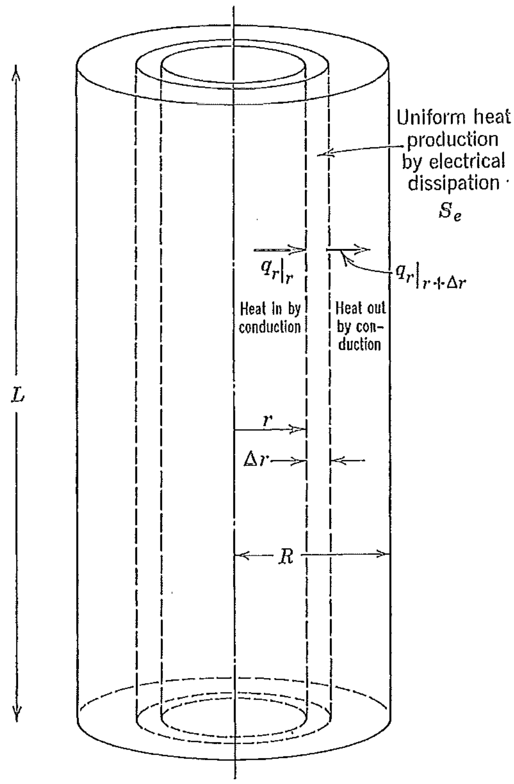


Fig. 9.2–1. Cylindrical shell over which energy balance is made in order to get temperature distribution in an electrically heated wire.

When k is assumed to be constant, this first-order differential equation may be integrated to give

$$T = -\frac{S_e r^2}{4k} + C_2 \tag{9.2–11}$$

The integration constant C_2 is determined from

B.C. 2: at $r = R$ $T = T_0$ (9.2–12)

Hence C_2 is found to be $T_0 + (S_e R^2/4k)$ and Eq. 9.2–11 becomes

$$\boxed{T - T_0 = \frac{S_e R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right]} \tag{9.2–13}$$